Mark Scheme (Results)

Summer 2023

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F3 (WFM03)
Paper 01

Question Number	Scheme	Notes	Marks
1	$7\cosh x+3\sinh x=2e^{x}+7\Rightarrow$ $7\left(\frac{e^{x}+e^{-x}}{2}\right)+3\left(\frac{e^{x}-e^{-x}}{2}\right)=2e^{x}+7$ $\left\{\frac{7}{2}e^{x}+\frac{7}{2}e^{-x}+\frac{3}{2}e^{x}-\frac{3}{2}e^{-x}=2e^{x}+7\right\}$	Substitutes at least one correct exponential form for either of the hyperbolic terms and achieves an **equation** in exponentials and constants alone	M1
	$\Rightarrow 7\left(e^{2x}+1\right)+3\left(e^{2x}-1\right)=4e^{2x}+14e^{x}$ $\left\{\Rightarrow 5e^{2x}+2=2e^{2x}+7e^{x}\right\}$	Multiplies through by e^{x} to obtain any equation that would form a 3TQ in e^{x} if like terms were collected	M1
	$\Rightarrow 6e^{2x}-14e^{x}+4=0\quad\left\{3e^{2x}-7e^{x}+2=0\right\}$	A correct three term quadratic in e^{x}. Could be implied by a correct root even if terms have not been collected.	A1
	$\Rightarrow\left(3e^{x}-1\right)\left(e^{x}-2\right)=0\Rightarrow e^{x}=...$	Solves their 3TQ - usual rules. One correct root for their quadratic if no working. Ignore labelling of the roots even if e.g.,"x" is used.	M1
	$x=\ln 2,\ \ln\frac{1}{3}$	Both correct and simplified but do not isw if there are **other answers.** Allow $-\ln\frac{1}{2}$ for ln 2 and –ln 3 or ln 3^{-1} for $\ln\frac{1}{3}$	A1
	Answer only is 0/5		**Total 5**
	Note that it is possible to multiply through by e^{-x} to form an equation in e^{-2x}, e^{-x} and constants. Score as main scheme, e.g., $\frac{7}{2}e^{x}+\frac{7}{2}e^{-x}+\frac{3}{2}e^{x}-\frac{3}{2}e^{-x}=2e^{x}+7$ $\Rightarrow\frac{7}{2}+\frac{7}{2}e^{-2x}+\frac{3}{2}-\frac{3}{2}e^{-2x}=2+7e^{-x}$ (M1) $\Rightarrow 2e^{-2x}-7e^{-x}+3=0$ (A1) $\left(2e^{-x}-1\right)\left(e^{-x}-3\right)=0\Rightarrow e^{-x}=\frac{1}{2},\ 3$ (M1) $\Rightarrow e^{x}=2,\ \frac{1}{3}\ \Rightarrow x=\ln 2,\ \ln\frac{1}{3}$ (A1)		

Question Number	Scheme	Notes	Marks
2	Condone poor notation e.g., determinant lines used for matrix bracketing		
(a)	$\det\begin{pmatrix}2&0&0\\0&1&4\\3&-2&-3\end{pmatrix}\ \left\{=2\times(-3+8)\right\}\ =10$	Correct value for determinant, seen or stated and not just in a final answer	B1
	$\left\{\text{Minors}:\begin{pmatrix}5&-12&-3\\0&-6&-4\\0&8&2\end{pmatrix}\Rightarrow\right\}\ \text{Cofactors}:\begin{pmatrix}5&12&-3\\0&-6&4\\0&-8&2\end{pmatrix}$	Attempts the cofactor matrix with at least 6 correct elements	M1
	Inverse is $\dfrac{1}{\text{"10"}}\begin{pmatrix}5&0&0\\12&-6&-8\\-3&4&2\end{pmatrix}$ or e.g., $\begin{pmatrix}\frac{1}{2}&0&0\\\frac{6}{5}&-\frac{3}{5}&-\frac{4}{5}\\-\frac{3}{10}&\frac{2}{5}&\frac{1}{5}\end{pmatrix}$	Correct inverse but allow ft on their "10". Allow equivalent fractions/decimals. A0 if clearly obtained incorrectly	A1ft
	Work to obtain Adj(**M**) must be seen but it may be minimal, e.g., sight of the matrix of minors followed by the correct answer is acceptable. Note that B0 M1 A1 is possible.		**(3)**
(b)	$\dfrac{1}{10}\begin{pmatrix}5&0&0\\12&-6&-8\\-3&4&2\end{pmatrix}\begin{pmatrix}u\\v\\w\end{pmatrix}=\ldots$	Multiplies their $\mathbf{M}^{-1}$ by $\begin{pmatrix}u\\v\\w\end{pmatrix}$ Must use a matrix other than **M** – not just changed by application of determinant. Condone sight of $\mathbf{vM}^{-1}=\ldots$ but must not be a clearly incorrect multiplication method	M1
	$\begin{pmatrix}x\\y\\z\end{pmatrix}=\dfrac{1}{10}\begin{pmatrix}5u\\12u-6v-8w\\-3u+4v+2w\end{pmatrix}$ or $\begin{pmatrix}\frac{1}{2}u\\\frac{6}{5}u-\frac{3}{5}v-\frac{4}{5}w\\-\frac{3}{10}u+\frac{2}{5}v+\frac{1}{5}w\end{pmatrix}$ or $\dfrac{1}{d}\begin{pmatrix}5u\\12u-6v-8w\\-3u+4v+2w\end{pmatrix}$ or $\begin{pmatrix}\frac{5}{d}u\\\frac{12}{d}u-\frac{6}{d}v-\frac{8}{d}w\\-\frac{3}{d}u+\frac{4}{d}v+\frac{2}{d}w\end{pmatrix}$ A1ft: Two correct vector components, coordinates or equations, ft their $d\neq 0$ A1ft: All three correct ft their non-zero $d\neq 0$ Must be exact (and not rounded decimals for ft) **These ft marks are not available for an incorrect** Adj(**M**)		A1ft A1ft
			(3)
Alt Using **M**	$2x=u \qquad x=\ldots$ $y+4z=v \ \Rightarrow\ y=\ldots$ $3x-2y-3z=w \qquad z=\ldots$	Uses $\mathbf{M}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}u\\v\\w\end{pmatrix}$ and finds x, y and z as functions of u, v and w Condone sight of $\mathbf{vM}=\ldots$ but must not be a clearly incorrect multiplication method	M1
	$x=\frac{1}{2}u$ $y=\frac{6}{5}u-\frac{3}{5}v-\frac{4}{5}w$ $z=-\frac{3}{10}u+\frac{2}{5}v+\frac{1}{5}w$	A1: Two correct equations A1: All three correct Any form with terms collected	A1 A1
			(3)

Question Number	Scheme	Notes	Marks
2(c)	$3x-7y+2z=-3 \Rightarrow 3\left(\frac{1}{2}u\right)-7\left(\frac{6}{5}u-\frac{3}{5}v-\frac{4}{5}w\right)+2\left(-\frac{3}{10}u+\frac{2}{5}v+\frac{1}{5}w\right)=-3$	Substitutes their expressions into the equation for Π_1	M1
	$-15u+10v+12w=-6$	Correct **equation.** Terms in any order but constant isolated. Accept any integer multiples.	A1
			(2)
			Total 8
Alts	To gain any marks by an alternative approach, a complete attempt at a Cartesian equation for Π_2 must be made by a viable strategy e.g.,		
	general point on $3x-7y+2z=-3$ is $\left(s,\ t,\ -\frac{3}{2}s+\frac{7}{2}t-\frac{3}{2}\right)$ $\begin{pmatrix}2&0&0\\0&1&4\\3&-2&-3\end{pmatrix}\begin{pmatrix}s\\t\\-\frac{3}{2}s+\frac{7}{2}t-\frac{3}{2}\end{pmatrix} \Rightarrow \begin{matrix}u=2s\\v=-6s+15t-6\\w=\frac{15}{2}s-\frac{25}{2}t+\frac{9}{2}\end{matrix} \Rightarrow \begin{matrix}v=-3u+15t-6\\t=-\frac{2}{25}\left(w-\frac{15}{2}\left(\frac{u}{2}\right)-\frac{9}{2}\right)\end{matrix}$ $\Rightarrow v=-3u-\frac{6}{5}w+\frac{9}{2}u+\frac{27}{5}-6$ Obtains a plane equation in any Cartesian form		M1
	$\left\{v=\frac{3}{2}u-\frac{6}{5}w-\frac{3}{5}\Rightarrow\right\}$ $-15u+10v+12w=-6$	Correct **equation.** Terms in any order but constant isolated. Accept any integer multiples.	A1
			(2)
			Total 8

Question Number	Scheme	Notes	Marks
3(a) Way 1 Identities first then squares	$y=\frac{1}{2}\left(\tan x+\cot x\right)\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}\left(\sec^2 x-\mathrm{cosec}^2 x\right)$ oe	Correct derivative. Any equivalent.	B1
	$=\frac{1}{2}\left(1+\tan^2 x-\left(1+\cot^2 x\right)\right)$ $\left\{=\frac{1}{2}\left(\tan^2 x-\cot^2 x\right)\right\}$	Applies $\sec^2 x=\pm\tan^2 x\pm 1$ and $\mathrm{cosec}^2 x=\pm\cot^2 x\pm 1$ to their derivative	M1
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=\frac{1}{4}\left(\tan^4 x+\cot^4 x-2\tan^2 x\cot^2 x\right)$	Squares to a 3 term expression (or 4 if middle terms uncollected) $2\tan^2 x\cot^2 x$ can be seen as 2 **Requires previous M mark.**	**d**M1
	$\left\{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=1+\frac{1}{4}\left(\tan^4 x+\cot^4 x-2\right)\right\}$ $\Rightarrow\frac{1}{4}\left(\tan^4 x+\cot^4 x+2\right)$ or $\frac{1}{4}\tan^4 x+\frac{1}{4}\cot^4 x+\frac{1}{2}$ **Not implied. Must be seen**	Adds the 1 and achieves either expression shown but allow the constant to be multiplied by $\tan^2 x\cot^2 x$ May be seen as e.g., $\frac{1}{2}\sqrt{\tan^4 x+\cot^4 x+2\tan^2 x\cot^2 x}$	A1
	$s=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x=\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\left(\tan^2 x+\cot^2 x\right)\mathrm{d}x$ * Allow $\int\frac{1}{2}\left(\tan^2 x+\cot^2 x\right)$ or $\frac{1}{2}\int\ \tan^2 x+\cot^2 x$	M1: Applies the arc length formula with their $\frac{\mathrm{d}y}{\mathrm{d}x}$ A1: Correct result achieved with no clear mathematical errors seen. Condone omission of "dx" and/or limits and **occasional** missing arguments.	M1 A1*
	Converting to sin & cos: likely to score max of 100010 unless tan & cot are convincingly recovered		**(6)**
Way 2 Squares first then identities	$y=\frac{1}{2}\left(\tan x+\cot x\right)\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}\left(\sec^2 x-\mathrm{cosec}^2 x\right)$ oe	Correct derivative. Any equivalent.	B1
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=\frac{1}{4}\left(\sec^4 x+\mathrm{cosec}^4 x-2\sec^2 x\,\mathrm{cosec}^2 x\right)$	Squares a derivative **of the correct form** to obtain a 3 (or 4 if middle terms uncollected) term expression.	M1
	$=\frac{1}{4}\left(\left(1+\tan^2 x\right)^2+\left(1+\cot^2 x\right)^2-2\left(1+\tan^2 x\right)\left(1+\cot^2 x\right)\right)$ $\left\{=\frac{1}{4}\left(1+2\tan^2 x+\tan^4 x+1+2\cot^2 x+\cot^4 x-2-2\tan^2 x-2\cot^2 x-2\tan^2 x\cot^2 x\right)\right\}$	Applies $\sec^2 x=\pm\tan^2 x\pm 1$ **twice and** $\mathrm{cosec}^2 x=\pm\cot^2 x\pm 1$ **twice**. **Requires previous M mark.**	**d**M1
	$\left\{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=1+\frac{1}{4}\left(\tan^4 x+\cot^4 x-2\right)\right\}$ $\Rightarrow\frac{1}{4}\left(\tan^4 x+\cot^4 x+2\right)$ or $\frac{1}{4}\tan^4 x+\frac{1}{4}\cot^4 x+\frac{1}{2}$ **Not implied. Must be seen**	Adds the 1 and achieves either expression shown but allow the constant to be multiplied by $\tan^2 x\cot^2 x$ May be seen as e.g., $\frac{1}{2}\sqrt{\tan^4 x+\cot^4 x+2\tan^2 x\cot^2 x}$	A1
	$s=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x=\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\tan^2 x+\cot^2 x\,\mathrm{d}x$ * Allow $\int\frac{1}{2}\left(\tan^2 x+\cot^2 x\right)$ or $\frac{1}{2}\int\ \tan^2 x+\cot^2 x$	M1: Applies the arc length formula with their $\frac{\mathrm{d}y}{\mathrm{d}x}$ A1: Correct result achieved with no clear mathematical errors seen. Condone omission of "dx" and/or limits and **occasional** missing arguments.	M1 A1*
	Converting to sin & cos: likely to score max of 100010 unless tan & cot are convincingly recovered		**(6)**

Question Number	Scheme	Notes	Marks
3(b)	$\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\left(\tan^2 x+\cot^2 x\right)\,\mathrm{d}x=\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\left(\sec^2 x-1+\operatorname{cosec}^2 x-1\right)\,\mathrm{d}x$	Applies $\tan^2 x=\pm\sec^2 x\pm1$ and $\cot^2 x=\pm\operatorname{cosec}^2 x\pm1$ to the integral	M1
	Work in sin and cos must use identities (sign errors only) and lead to a result of the form below after integration condoning the absence of a term in *x* but allow the last M to be available following a completed attempt at integration.		
	$=\frac{1}{2}\left[\tan x-\cot x-2x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	M1: For $\pm\sec^2 x\to\pm\tan x$ and $\pm\operatorname{cosec}^2 x\to\pm\cot x$ **Requires previous M mark.** A1: Correct integration. Limits not required.	**d**M1 A1
	$\frac{1}{2}\left(\tan\frac{\pi}{3}-\cot\frac{\pi}{3}-\frac{2\pi}{3}-\left(\tan\frac{\pi}{6}-\cot\frac{\pi}{6}-\frac{2\pi}{6}\right)\right)$ $\left\{\frac{1}{2}\left(\sqrt{3}-\frac{2\pi}{3}-\frac{\sqrt{3}}{3}-\left(\frac{\sqrt{3}}{3}-\frac{\pi}{3}-\sqrt{3}\right)\right)\right\}$	Applies the limits (see note below) following any completed attempt at integration. Allow slips provided it is a clear attempt at $\mathrm{f}\left(\frac{\pi}{3}\right)-\mathrm{f}\left(\frac{\pi}{6}\right)$	M1
	Correct answer in any exact simplified form with 2 terms e.g. $\frac{1}{2}\left(\frac{4\sqrt{3}}{3}-\frac{\pi}{3}\right)$, $\frac{2\sqrt{3}}{3}-\frac{\pi}{6}$, $\frac{2}{\sqrt{3}}-\frac{\pi}{6}$, $\frac{1}{3}\left(2\sqrt{3}-\frac{\pi}{2}\right)$, $\frac{4\sqrt{3}-\pi}{6}$		A1
	Note they may apply the limits $\frac{\pi}{4}$ & $\frac{\pi}{6}$ or $\frac{\pi}{3}$ & $\frac{\pi}{4}$ and then double the result.		**(5)**
	Just the answer or decimal answer (0.6311017628) is 0/5		**Total 11**

Question Number	Scheme	Notes	Marks
4	Allow any suitable vector notation throughout this question.		
(a)	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} . \begin{pmatrix} -1 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix} . \begin{pmatrix} -1 \\ 3 \\ 3 \end{pmatrix} \Rightarrow ...$ or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} . \begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix} . \begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix} \Rightarrow ...$ $-x+3y+3z=-5$ and $2x-5z=16$	M1: Uses **r**.**n** = **a**.**n** at least once to obtain a plane equation A1: Both correct equations. Accept in **r**.**n** = p form	M1 A1
	e.g., $x = \dfrac{16+5z}{2}$	Obtains one variable (may be written as parameter for all marks) in terms of one of the other variables	M1
	$z = \dfrac{2x-16}{5} \Rightarrow x = 5+3y+3\left(\dfrac{2x-16}{5}\right)$ $\Rightarrow 5x = 25+15y+6x-48 \Rightarrow x = -15y+23$ $\left\{x = -15y+23 = \dfrac{16+5z}{2}\right\}$	M1: Obtains the variable/parameter in terms of the third variable (or the two other variables in terms of the parameter) A1: Both correct equations	M1 A1 **(M1 on epen)**
	Alternatively, $y = \dfrac{-x+23}{15} = \dfrac{6-z}{6}$ or $z = \dfrac{2x-16}{5} = 6-6y$		
	$\left\{\dfrac{x-0}{1} = \dfrac{y-\frac{23}{15}}{-\frac{1}{15}} = \dfrac{z+\frac{16}{5}}{\frac{2}{5}} \Rightarrow\right\}$ $\mathbf{r} = \begin{pmatrix} 0 \\ \frac{23}{15} \\ -\frac{16}{5} \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -\frac{1}{15} \\ \frac{2}{5} \end{pmatrix}$ M1: Attempts vector equation of line but "**r** =" may be missing. **Requires all previous M marks.** Allow numerical slips but it must be a correct method i.e., an attempt at $\Rightarrow \dfrac{x-x_1}{l} = \dfrac{y-y_1}{m} = \dfrac{z-z_1}{n} \Rightarrow \mathbf{r} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \lambda \begin{pmatrix} l \\ m \\ n \end{pmatrix}$ A1: Any correct equation including "**r** ="		**d**M1 A1
	Or $\left\{\dfrac{x-23}{-15} = \dfrac{y-0}{1} = \dfrac{z-6}{-6} \Rightarrow\right\}$ $\mathbf{r} = \begin{pmatrix} 23 \\ 0 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} -15 \\ 1 \\ -6 \end{pmatrix}$ or $\left\{\dfrac{x-8}{\frac{5}{2}} = \dfrac{y-1}{-\frac{1}{6}} = \dfrac{z-0}{1} \Rightarrow\right\}$ $\mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} \frac{5}{2} \\ -\frac{1}{6} \\ 1 \end{pmatrix}$		
	Note that the line may be given in $(\mathbf{r}-\mathbf{a})\times\mathbf{b} = 0$ or $\mathbf{r}\times\mathbf{b} = \mathbf{a}\times\mathbf{b}$ form		**(7)**

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4(a) **Alt** **Finds point and vector product of normals**	$\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}-1\\3\\3\end{pmatrix}=\begin{pmatrix}2\\4\\-5\end{pmatrix}.\begin{pmatrix}-1\\3\\3\end{pmatrix}\Rightarrow\ldots$ or $\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}2\\0\\-5\end{pmatrix}=\begin{pmatrix}3\\6\\-2\end{pmatrix}.\begin{pmatrix}2\\0\\-5\end{pmatrix}\Rightarrow\ldots$ $-x+3y+3z=-5$ and $2x-5z=16$	M1: Uses **r**.**n** = **a**.**n** at least once to obtain a plane equation A1: Both correct equations Accept in **r**.**n** = p form	M1 A1
	e.g., $x=0\Rightarrow z=-\frac{16}{5}$	Sets one variable equal to a value and finds a value for another variable. Correct for their equations if no working.	M1
	$3y=-5-3\left(-\frac{16}{5}\right)\Rightarrow y=\frac{23}{15}\ \left\{\Rightarrow\left(0,\ \frac{23}{15},\ -\frac{16}{5}\right)\right\}$ Or e.g., $(23,\ 0,\ 6)$, $(8,\ 1,\ 0)$ Points will have the form $(23-15\alpha,\ \alpha,\ 6-6\alpha)$	M1: Proceeds to find a value for the remaining variable. Correct for their equations if no working. A1: Correct values	M1 A1 **(M1 on epen)**
	$\begin{pmatrix}-1\\3\\3\end{pmatrix}\times\begin{pmatrix}2\\0\\-5\end{pmatrix}=\ldots\Rightarrow\mathbf{r}=\begin{pmatrix}0\\\frac{23}{15}\\-\frac{16}{5}\end{pmatrix}+\lambda\begin{pmatrix}-15\\1\\-6\end{pmatrix}$ $\left\{\mathbf{r}=\begin{pmatrix}23\\0\\6\end{pmatrix}+\lambda\begin{pmatrix}-15\\1\\-6\end{pmatrix}\quad \mathbf{r}=\begin{pmatrix}8\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}-15\\1\\-6\end{pmatrix}\right\}$	**d**M1: Attempts vector product of normals (two correct components if method unclear) and forms vector equation with **point** and direction in correct places but allow for a copying error or mix up with components. **Note** that they could obtain the direction from 2 points on the line. **Requires all previous M marks.** "**r** =" may be missing. A1: Any correct equation including "**r** ="	**d**M1 A1
			(7)

Question Number	Scheme	Notes	Marks
4(b)	Note: If 0/5 allow SC 00010 for a correct volume formula seen **<u>for tetrahedron *ABCD*</u>** e.g., $\frac{1}{6}\left\lvert\overrightarrow{CD}.\left(\overrightarrow{CA}\times\overrightarrow{CB}\right)\right\rvert$ Allow with missing modulus but not vector arrows unless implied by further work.		
Way 1 STP inc. $\overrightarrow{CD}$	$\left\lvert\begin{pmatrix}-15\\1\\-6\end{pmatrix}\right\rvert=\sqrt{262}\Rightarrow\overrightarrow{CD}=\frac{5}{\sqrt{262}}\begin{pmatrix}-15\\1\\-6\end{pmatrix}$	Attempts magnitude (allow numerical slip) of their direction vector and scales correctly to length 5	M1
	Let C be the point $(8, 1, 0)$ $\overrightarrow{CA}=\begin{pmatrix}2\\4\\-5\end{pmatrix}-\begin{pmatrix}8\\1\\0\end{pmatrix}=\ldots\left\{\begin{pmatrix}-6\\3\\-5\end{pmatrix}\right\}$ and $\overrightarrow{CB}=\begin{pmatrix}3\\6\\-2\end{pmatrix}-\begin{pmatrix}8\\1\\0\end{pmatrix}=\ldots\left\{\begin{pmatrix}-5\\5\\-2\end{pmatrix}\right\}$	Finds vectors for **any** two edges other than *CD*. Could be implied by a distance calculation **if *C* and/or *D* defined**. This mark is not scored if either vector is in terms of a parameter unless it is assigned a value (or is eliminated appropriately) later.	M1
	$\overrightarrow{CD}.\left(\overrightarrow{CA}\times\overrightarrow{CB}\right)=\frac{5}{\sqrt{262}}\begin{pmatrix}-15\\1\\-6\end{pmatrix}.\begin{pmatrix}-6\\3\\-5\end{pmatrix}\times\begin{pmatrix}-5\\5\\-2\end{pmatrix}=\ldots\quad\left\{=-\frac{910}{\sqrt{262}}\right\}$	Uses an appropriate scalar triple product with their vectors and finds a value. **Must not include position vectors**. Could be inexact. M0 if clear evidence of an inappropriate method	M1
	$V=\frac{1}{6}\left\lvert\overrightarrow{CD}.\left(\overrightarrow{CA}\times\overrightarrow{CB}\right)\right\rvert=\ldots=\frac{455}{3\sqrt{262}}$ or $\frac{455\sqrt{262}}{786}$	**d**M1: Divides their STP result by 6 and obtains a positive value. Could be inexact. Modulus might not be seen. **Requires previous M mark.** A1: A correct **exact** value	**d**M1 A1
			(5)
Way 2 STP not inc. $\overrightarrow{CD}$	$\left\lvert\begin{pmatrix}-15\\1\\-6\end{pmatrix}\right\rvert=\sqrt{262}\Rightarrow\overrightarrow{CD}=\frac{5}{\sqrt{262}}\begin{pmatrix}-15\\1\\-6\end{pmatrix}$	Attempts magnitude (allow numerical slip) of their direction vector and scales correctly to length 5	M1
	Let C be the point $(8, 1, 0)$ $\overrightarrow{AC}=\begin{pmatrix}8\\1\\0\end{pmatrix}-\begin{pmatrix}2\\4\\-5\end{pmatrix}=\ldots\left\{\begin{pmatrix}6\\-3\\5\end{pmatrix}\right\}$ and $\overrightarrow{AB}=\begin{pmatrix}3\\6\\-2\end{pmatrix}-\begin{pmatrix}2\\4\\-5\end{pmatrix}=\ldots\left\{\begin{pmatrix}1\\2\\3\end{pmatrix}\right\}$	Finds vectors for **any** two edges other than *CD*. Could be implied by a distance calculation **if *C* and/or *D* defined**. (See also comment for second M1 in Way 1 re use of a parameter)	M1
	$\overrightarrow{OD}=\begin{pmatrix}8\\1\\0\end{pmatrix}+\frac{5}{\sqrt{262}}\begin{pmatrix}-15\\1\\-6\end{pmatrix}\Rightarrow\overrightarrow{AD}=\begin{pmatrix}\frac{-75}{\sqrt{262}}+8\\\frac{5}{\sqrt{262}}+1\\\frac{-30}{\sqrt{262}}\end{pmatrix}-\begin{pmatrix}2\\4\\-5\end{pmatrix}=\begin{pmatrix}\frac{-75}{\sqrt{262}}+6\\\frac{5}{\sqrt{262}}-3\\\frac{-30}{\sqrt{262}}+5\end{pmatrix}$ $\Rightarrow\overrightarrow{AD}.\left(\overrightarrow{AB}\times\overrightarrow{AC}\right)=\begin{pmatrix}\frac{-75}{\sqrt{262}}+6\\\frac{5}{\sqrt{262}}-3\\\frac{-30}{\sqrt{262}}+5\end{pmatrix}.\begin{pmatrix}1\\2\\3\end{pmatrix}\times\begin{pmatrix}6\\-3\\5\end{pmatrix}=\ldots\quad\left\{=-\frac{910}{\sqrt{262}}\right\}$	Uses an appropriate scalar triple product with their vectors and finds a value. **Must not include position vectors**. Could be inexact. M0 if clear evidence of an inappropriate method	M1
	$V=\frac{1}{6}\left\lvert\overrightarrow{AD}.\left(\overrightarrow{AB}\times\overrightarrow{AC}\right)\right\rvert=\ldots=\frac{455}{3\sqrt{262}}$ or $\frac{455\sqrt{262}}{786}$	**d**M1: Divides their STP result by 6 and obtains a positive value. Could be inexact. Modulus might not be seen. **Requires previous M mark.** A1: A correct **exact** value	**d**M1 A1
			(5)

Question Number	Scheme	Notes	Marks
4(b) Way 3 Triangle area + perp. distance to plane & vol. of pyramid	$\left\lvert\begin{pmatrix}-15\\1\\-6\end{pmatrix}\right\rvert=\sqrt{262}\Rightarrow\overrightarrow{CD}=\frac{5}{\sqrt{262}}\begin{pmatrix}-15\\1\\-6\end{pmatrix}$	Attempts magnitude of their direction vector and scales to length 5. See note after next M below.	M1
	Let C be the point $(8, 1, 0)$ $Area\ \Delta ACD=\frac{1}{2}\left\lvert\overrightarrow{CD}\times\overrightarrow{CA}\right\rvert=\frac{1}{2}\left\lvert\frac{5}{\sqrt{262}}\begin{pmatrix}-15\\1\\-6\end{pmatrix}\times\begin{pmatrix}-6\\3\\-5\end{pmatrix}\right\rvert=...\ \left\{=\frac{65\sqrt{19}}{2\sqrt{262}}\right\}$ Uses formula to find a value for the area of one of the faces. Must be a full method (vector product and modulus). Condone missing $\frac{1}{2}$ Any attempts by trig/Pythagoras must be complete and credible		M1
	Note: It is possible to obtain the area of a relevant triangle such as ACD by e.g., finding the length of the perpendicular distance of point A to the line and multiplying this by $\frac{1}{2}$ x 5 – in such cases allow the first M for completing a viable attempt at the height of the triangle and the second for the area (Condone missing $\frac{1}{2}$)		
	ΔACD is in Π_1 so perp. height of tetrahedron is shortest dist. of $B(3, 6, -2)$ to $-x+3y+3z=-5$: $\left\lvert\frac{-1\times3+3\times6+3\times(-2)+5}{\sqrt{(-1)^2+3^2+3^2}}\right\rvert=...\ \left\{\frac{14}{\sqrt{19}}\right\}$	Obtains a value for the perpendicular height via formula or any credible method (examples below)	M1
	Parallel planes: $\begin{pmatrix}3\\6\\-2\end{pmatrix}.\begin{pmatrix}-1\\3\\3\end{pmatrix}=9,\ \begin{pmatrix}2\\4\\-5\end{pmatrix}.\begin{pmatrix}-1\\3\\3\end{pmatrix}=-5\Rightarrow\left\lvert\frac{-5-9}{\sqrt{(-1)^2+3^2+3^2}}\right\rvert=\frac{14}{\sqrt{19}}$ Projection/Resolving: $\overrightarrow{BA}=\begin{pmatrix}1\\2\\3\end{pmatrix}\Rightarrow\frac{\begin{pmatrix}1\\2\\3\end{pmatrix}.\begin{pmatrix}-1\\3\\3\end{pmatrix}}{\sqrt{(-1)^2+3^2+3^2}}=\frac{14}{\sqrt{19}}$		
	$V=\frac{1}{3}\times\frac{65\sqrt{19}}{2\sqrt{262}}\times\frac{14}{\sqrt{19}}=...=\frac{455}{3\sqrt{262}}$ or $\frac{455\sqrt{262}}{786}$	M1: Uses $\frac{1}{3}\times$area $\Delta\times$perp. height and obtains a positive value. $\frac{1}{2}$ must have been used for triangle area earlier unless they now use $\frac{1}{6}\times...$ **Requires previous M mark**. A1: Either correct **exact** value	**d**M1 A1
			(5)
			Total 12

Question Number	Scheme	Notes	Marks
5	$\mathbf{M}=\begin{pmatrix}1&2&k\\-1&-3&4\\2&6&-8\end{pmatrix}$		
(i) & (ii) Mark the parts together	$\det\begin{pmatrix}1-\lambda&2&k\\-1&-3-\lambda&4\\2&6&-8-\lambda\end{pmatrix}$ $=\pm\left[(1-\lambda)\left((-3-\lambda)(-8-\lambda)-24\right)-2\left((-1)(-8-\lambda)-8\right)+k\left((-1)(6)-2(-3-\lambda)\right)\right]$	Recognisable complete attempt at $\det\left(\mathbf{M}-\lambda\mathbf{I}\right)$. May use other rows/columns. Allow $\pm$ and slips including +2 for first –2	M1
	Sarrus $\Rightarrow\pm[(1-\lambda)(-3-\lambda)(-8-\lambda)+(2)(4)(2)+(k)(-1)(6)-(k)(-3-\lambda)(2)-(1-\lambda)(4)(6)-(2)(-1)(-8-\lambda)]$		
	$=(1-\lambda)\left(\lambda^2+11\lambda\right)-2\lambda+2k\lambda$ $=-\lambda^3-10\lambda^2+9\lambda+2k\lambda$ $=\lambda\left(-\lambda^2-10\lambda+9+2k\right)$	M1: Obtains $\{\lambda\}\left(a\lambda^2+b\lambda+c+dk \text{ oe}\right)$ $a, b, c, d\neq 0$ A1: Correct expression – allow: $\pm\{\lambda\}\left(-\lambda^2-10\lambda+9+2k \text{ oe}\right)$ or $\pm\{\lambda\}\left(\lambda^2+10\lambda-9-2k \text{ oe}\right)$ Allow quadratic to be unsimplified and the marks can be implied if the initial λ has been removed	M1 A1
	{One eigenvalue is zero, if repeated then} $9+2k=0\Rightarrow k=...$ **or** $\left\{\pm\left(-\lambda^2-10\lambda+9+2k\right)\text{has repeated roots so}\right\}$ $b^2-4ac=0\Rightarrow\begin{cases}100-4(-1)(9+2k)=0\\100-4(1)(-9-2k)=0\end{cases}\Rightarrow k=...$	Attempts to set their $c+dk=0$ and solves for k **or** Considers the case of their quadratic $a\lambda^2+b\lambda+c+dk=0$ having a repeated root and uses a valid strategy to find k	M1
	Alternative approaches with $\lambda^2+10\lambda-9-2k=0$: $\left(\lambda+a\right)^2=\lambda^2+2a\lambda+a^2\Rightarrow 2a=10\Rightarrow -9-2k=5^2\Rightarrow k=...$ sum of roots $=-10\Rightarrow$ root $=-5\Rightarrow$ product of roots $=(-5)^2=-9-2k\Rightarrow k=...$		
	$k=-\dfrac{9}{2}$ or $k=-17$	One correct value for k	A1
	{One eigenvalue is zero, if repeated then} $9+2k=0\Rightarrow k=...$ **and** $\left\{\pm\left(-\lambda^2-10\lambda+9+2k\right)\text{has repeated roots so}\right\}$ $b^2-4ac=0\Rightarrow\begin{cases}100-4(-1)(9+2k)=0\\100-4(1)(-9-2k)=0\end{cases}\Rightarrow k=...$	Attempts to set their $c+dk=0$ and solves for k **and** Considers the case of their quadratic $a\lambda^2+b\lambda+c+dk=0$ having a repeated root and uses a valid strategy to find k	M1
	$k=-\dfrac{9}{2}$ with eigenvalue –10 {and 0 repeated} $k=-17$ with eigenvalue –5 {repeated and 0}	Both correct values of k and the associated non-zero eigenvalues clearly assigned. No additional eigenvalues or values for k	A1
			Total 7

Question Number	Scheme	Notes	Marks
	$\frac{x^2}{16}+\frac{y^2}{9}=1 \qquad P\left(4\cos\theta,\, 3\sin\theta\right)$		
6(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{3\cos\theta}{4\sin\theta}$ or $\frac{2x}{16}+\frac{2y}{9}\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{18x}{32y}$ or $\frac{x^2}{16}+\frac{y^2}{9}=1\Rightarrow y=3\left(1-\frac{x^2}{16}\right)^{\frac{1}{2}}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{2}\left(1-\frac{x^2}{16}\right)^{-\frac{1}{2}}\times-\frac{2x}{16}$	Uses a correct method and finds an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ of the correct form (sign and coefficient slips only)	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{3\cos\theta}{4\sin\theta}$ oe e.g. $-\frac{3}{4}\cot\theta$ oe	Any correct derivative in terms of θ only.	A1
	$y-3\sin\theta=-\frac{3\cos\theta}{4\sin\theta}\left(x-4\cos\theta\right)$ or or $y=-\frac{3\cos\theta}{4\sin\theta}x+c\Rightarrow 3\sin\theta=-\frac{3\cos\theta}{4\sin\theta}4\cos\theta+c$ $\Rightarrow c=...\ \left\{\frac{12\sin^2\theta+12\cos^2\theta}{4\sin\theta}\right\}$	Applies correct straight line method using any gradient in terms of θ. If they use $y=mx+c$ they must substitute coordinates correctly and reach $c=...$ M0 if use normal gradient	M1
	$\Rightarrow 4y\sin\theta-12\sin^2\theta=-3x\cos\theta+12\cos^2\theta$ or using $y=mx+c: y=-\frac{3\cos\theta}{4\sin\theta}x+\frac{3}{\sin\theta}\Rightarrow 4y\sin\theta=-3x\cos\theta+12$ $\Rightarrow 3x\cos\theta+4y\sin\theta\left\{=12(\cos^2\theta+\sin^2\theta)\right\}=12$ M1: Multiplies through to remove fraction to obtain an equation with trig expressions in sin and cos only. Allow this mark if they go straight to the given answer from a correct equation. Can score from use of a normal gradient and/or with coordinates wrongly placed but there must have been an attempt at a line. A1*: Correct equation from correct work. $\sin^2\theta$ and $\cos^2\theta$ must be seen somewhere in the working. Accept e.g., $\sin^2\theta+\cos^2\theta=1$ seen in side-working		M1 A1*
			(5)
(b)	$y-3\sin\theta=\frac{4\sin\theta}{3\cos\theta}\left(x-4\cos\theta\right)$ oe e.g., $4x\sin\theta-3y\cos\theta=7\sin\theta\cos\theta$ or $y=\frac{4\sin\theta}{3\cos\theta}x+c$ $\Rightarrow 3\sin\theta=\frac{4\sin\theta}{3\cos\theta}4\cos\theta+c\Rightarrow c=...\ \left\{\frac{-7\sin\theta\cos\theta}{3\cos\theta}\right\}$	M1: Applies correct straight line method with the negative reciprocal of their tangent gradient. If $y=mx+c$ is used coordinates must be substituted correctly and $c=...$ reached A1: Any correct equation	M1 A1
			(2)

Question Number	Scheme	Notes	Marks
6(c)	A is $\left(\dfrac{4}{\cos\theta}, 0\right)$	Any correct x-axis intercept of the tangent. Allow e.g., $\{x=\}\dfrac{12}{3\cos\theta}$, $4\sec\theta$ Could be on a diagram or implied by midpoint	B1
	$x=0 \Rightarrow y-3\sin\theta=-\dfrac{16}{3}\sin\theta \Rightarrow B$ is $\left(0,-\dfrac{7}{3}\sin\theta\right)$	Sets $x = 0$ in their **normal** equation (changed gradient) and finds y. Could be implied. Allow just $-\dfrac{7}{3}\sin\theta$ oe	M1
	So midpoint M of AB is $\left(\dfrac{2}{\cos\theta},-\dfrac{7}{6}\sin\theta\right)$	Any correct midpoint. Accept any equivalents and as $x=...,\ y=...$	A1
	$\sin^2\theta+\cos^2\theta=1 \Rightarrow \left(-\dfrac{6}{7}y\right)^2+\left(\dfrac{2}{x}\right)^2=1$	Uses $\sin^2\theta+\cos^2\theta=1$ to obtain an equation in x and y only. May follow incorrect or no attempt at midpoint	M1
	$\Rightarrow \dfrac{36}{49}y^2+\dfrac{4}{x^2}=1 \Rightarrow 36x^2y^2+49\times4=49x^2$ $\Rightarrow x^2\left(49-36y^2\right)=196$	**d**M1: Rearranges to the form $x^2\left(p \pm qy^2\right)=r,\quad p,\ q,\ r \in \mathbb{Z}$ **Requires all previous M marks**. A1: Correct equation	**d**M1 A1
			(6)
	Note that is possible to use e.g., $1+\tan^2\theta=\sec^2\theta$, for example: $M\left(2\sec\theta, \dfrac{-7\tan\theta}{6\sec\theta}\right) \Rightarrow \sec\theta=\dfrac{x}{2},\ y=\dfrac{-7\tan\theta}{3x} \Rightarrow \tan\theta=\dfrac{-3xy}{7} \Rightarrow 1+\dfrac{9x^2y^2}{49}=\dfrac{x^2}{4}$ (2nd M1) $\Rightarrow 1+\dfrac{9x^2y^2}{49}=\dfrac{x^2}{4} \Rightarrow 196+36x^2y^2=49x^2 \Rightarrow x^2\left(49-36y^2\right)=196$ (3rd M1, A1)		**Total 13**

Question Number	Scheme	Notes	Marks
7(a) **Way 1**	$I_n = \int \cosh^n 2x \, dx = \int \cosh 2x \cosh^{n-1} 2x \, dx$ $= \frac{1}{2}\sinh 2x \cosh^{n-1} 2x - \int \frac{1}{2}\sinh 2x \times (n-1)\cosh^{n-2} 2x \times 2\sinh 2x \, dx$	M1: Correct split and attempts to apply parts to obtain an expression of the correct form (sign and coefficient errors only). A1: Any correct expression	M1 A1
	$\left\{= \frac{1}{2}\sinh 2x \cosh^{n-1} 2x - (n-1)\int \sinh^2 2x \cosh^{n-2} 2x \, dx\right\}$ $= \frac{1}{2}\sinh 2x \cosh^{n-1} 2x - (n-1)\int \left(\cosh^2 2x - 1\right)\cosh^{n-2} 2x \, dx$	Applies $\sinh^2 2x = \pm\cosh^2 2x \pm 1$ **Requires previous M mark.**	**d**M1
	$\Rightarrow I_n = \frac{1}{2}\sinh 2x \cosh^{n-1} 2x - (n-1)\left(I_n - I_{n-2}\right)$	Introduces I_n and I_{n-2} – not implied by given answer. **Requires previous M mark.**	**dd**M1
	$\left\{\Rightarrow nI_n = \frac{1}{2}\sinh 2x \cosh^{n-1} 2x + (n-1)I_{n-2}\right\}$ $I_n = \frac{\sinh 2x \cosh^{n-1} 2x}{2n} + \frac{n-1}{n}I_{n-2}$ *	Fully correct proof. Condone missing 'dx's. Poor bracketing must be recovered before given answer but no other errors e.g., sin for sinh, or wrong or missing arguments	A1*
	Accept e.g., $I_n = \frac{(n-1)I_{n-2}}{n} + \frac{1}{2n}\sinh 2x \cosh^{n-1} 2x$		**(5)**
Way 2	$I_n = \int \cosh^n 2x \, dx = \int \cosh^2 2x \cosh^{n-2} 2x \, dx$ $= \int \left(\sinh^2 2x + 1\right)\cosh^{n-2} 2x \, dx$	M1: Correct split and applies $\sinh^2 2x = \pm\cosh^2 2x \pm 1$ to obtain an expression of the correct form (sign and coefficient errors only). A1: Correct expression	M1 A1
	$\left\{= \int \cosh^{n-2} 2x \, dx + \int \sinh^2 2x \cosh^{n-2} 2x \, dx\right\}$ $\int \sinh^2 2x \cosh^{n-2} 2x \, dx \left\{= \int \sinh 2x \cosh^{n-2} 2x \sinh 2x \, dx\right\}$ $= \frac{1}{2\left(n-1\right)}\sinh 2x \cosh^{n-1} 2x - \frac{1}{n-1}\int \cosh^n 2x \, dx$	Attempts to apply parts to obtain an expression of the correct form for $\int \sinh^2 2x \cosh^{n-2} 2x \, dx$ **Requires previous M mark.**	**d**M1
	$\Rightarrow I_n = I_{n-2} + \frac{1}{2\left(n-1\right)}\sinh 2x \cosh^{n-1} 2x - \frac{1}{n-1}I_n$	Introduces I_n and I_{n-2} – not implied by given answer. **Requires previous M mark.**	**dd**M1
	$\left\{\Rightarrow \left(n-1\right)I_n = \frac{1}{2}\sinh 2x \cosh^{n-1} 2x + (n-1)I_{n-2} - I_n\right\}$ $I_n = \frac{\sinh 2x \cosh^{n-1} 2x}{2n} + \frac{n-1}{n}I_{n-2}$ *	Fully correct proof. Condone missing 'dx's. Poor bracketing must be recovered before given answer but no other errors e.g., sin for sinh, or wrong or missing arguments	A1*
	Accept e.g., $I_n = \frac{(n-1)I_{n-2}}{n} + \frac{1}{2n}\sinh 2x \cosh^{n-1} 2x$		**(5)**

Question Number	Scheme	Notes	Marks
7(b)	$\left(1+\cosh 2x\right)^3 = 1+3\cosh 2x+3\cosh^2 2x+\cosh^3 2x$ Correct expansion. Could be implied e.g. by $x+3I_1+3I_2+I_3$ and allow if correct but terms are not collected. Condone if partially or completely in "x" provided terms <u>are</u> collected		B1
	$\int \cosh^2 2x \, dx$ or $I_2 = \frac{1}{4}\sinh 2x\cosh 2x + \frac{1}{2}I_0$ or $\int \cosh^3 2x \, dx$ or $I_3 = \frac{1}{6}\sinh 2x\cosh^2 2x + \frac{2}{3}I_1$	Completes an attempt to apply the reduction formula for I_2 **or** I_3. May be slips but must get two terms. May be seen with I_0 / I_1 attempted and/or embedded in expression for $\int \left(1+\cosh 2x\right)^3 dx$	M1
	$I_0 = x \qquad I_1 = \frac{1}{2}\sinh 2x$ $\int \left(1+\cosh 2x\right)^3 dx = \int \left(1+3\cosh 2x\right)dx + 3I_2 + I_3 =$ $x+\frac{3}{2}\sinh 2x+\frac{3}{4}\sinh 2x\cosh 2x+\frac{3}{2}x+\frac{1}{6}\sinh 2x\cosh^2 2x+\frac{1}{3}\sinh 2x\left(+c\right)$	$I_0 = x$ **and** $I_1 = \pm k\sinh 2x$ (condone I_1 from formula) **and** $\int \left(1+3\cosh 2x\right)dx \to x \pm q\sinh 2x$ **and** uses the above to obtain an expression for $\int \left(1+\cosh 2x\right)^3 dx$ **Requires previous M mark.**	**d**M1
	Note: **One** of I_2 and I_3 may be attempted directly – if so, correct identities must be used and an expression of a correct form obtained. Examples: $I_2 = \int \cosh^2 2x \, dx = \int \left(\frac{1}{2}\cosh 4x + \frac{1}{2}\right)dx = \frac{1}{8}\sinh 4x + \frac{x}{2}$ $\Rightarrow x+\frac{3}{2}\sinh 2x+\frac{3}{8}\sinh 4x+\frac{3}{2}x+\frac{1}{6}\sinh 2x\cosh^2 2x+\frac{1}{3}\sinh 2x\left(+c\right)$ $I_3 = \int \cosh^3 2x \, dx = \int \cosh 2x\left(\sinh^2 2x+1\right)dx = \frac{1}{6}\sinh^3 2x + \frac{1}{2}\sinh 2x$ $\Rightarrow x+\frac{3}{2}\sinh 2x+\frac{3}{4}\sinh 2x\cosh 2x+\frac{3}{2}x+\frac{1}{6}\sinh^3 2x+\frac{1}{2}\sinh 2x\left(+c\right)$ If exponential definitions are used they must be correct.		
	$=\frac{5}{2}x+\frac{11}{6}\sinh 2x+\frac{3}{4}\sinh 2x\cosh 2x+\frac{1}{6}\sinh 2x\cosh^2 2x\left(+c\right)$	Correct answer. Award when a correct expression with collected like terms is seen.	A1
	I_2 attempted directly $\Rightarrow \frac{5}{2}x+\frac{11}{6}\sinh 2x+\frac{3}{8}\sinh 4x+\frac{1}{6}\sinh 2x\cosh^2 2x\left(+c\right)$ I_3 attempted directly $\Rightarrow \frac{5}{2}x+2\sinh 2x+\frac{3}{4}\sinh 2x\cosh 2x+\frac{1}{6}\sinh^3 2x\left(+c\right)$		**(4)**
	If identities are used before a correct answer is seen with like terms collected then the work must be correct		**Total 9**

Question Number	Scheme	Notes	Marks
8(a)	$\left\{\frac{dy}{dx}=\right\}\text{arcosh}\,5x+\frac{ax}{\sqrt{bx^2-1}}$ or $\text{arcosh}\,5x+\frac{cx}{\sqrt{x^2-d}}$ (M1) $\Rightarrow \text{arcosh}(5x)+\frac{5x}{\sqrt{25x^2-1}}$ (A1) M1: Differentiates to obtain expression of the correct form $a,\ b,\ c,\ d\neq 0$ A1: Correct differentiation. Any equivalent form.		M1 A1
			(2)
(b)	$\frac{d}{dx}\left(x\text{arcosh}(5x)\right)=\text{arcosh}(5x)+"\frac{5x}{\sqrt{25x^2-1}}"\Rightarrow\int\text{arcosh}(5x)\,dx=x\,\text{arcosh}(5x)-\int"\frac{5x}{\sqrt{25x^2-1}}"dx$ M1: Rearranges **their** answer to (a) correctly and integrates or uses the correct formula to apply parts to $1\times\text{arcosh}\,5x$ to obtain the above.		M1
	$\int\text{arcosh}(5x)\,dx=x\,\text{arcosh}(5x)-\int\frac{5x}{\sqrt{25x^2-1}}dx$ A1: **Correct** expression – but **see note below on limited ft**		A1 **(limited ft)**
	$=x\,\text{arcosh}(5x)-\frac{1}{5}\left(25x^2-1\right)^{\frac{1}{2}}\ (+c)$	M1: $\int\frac{Ax}{\sqrt{Bx^2-1}}dx\to C\left(Bx^2-1\right)^{\frac{1}{2}}$ A1: Fully correct expression with $x$arcosh($5x$) - see note below for limited ft	M1 A1 **(limited ft)**
	Note: Substitutions: $u=5x\Rightarrow\left(u^2-1\right)^{\frac{1}{2}}\Rightarrow\left[\frac{1}{5}\sqrt{u^2-1}\right]_{\frac{5}{4}}^{3}$ $\quad u=25x^2-1\Rightarrow\left[\frac{1}{5}\sqrt{u}\right]_{\frac{9}{16}}^{8}$ M1: Correct form A1: Fully correct expression with $x$arcosh($5x$)		
	A limited ft for **one** of the errors in (a) shown below applies for the first two A marks. **However also allow the following if this error occurs in part (b)** which is most likely to come from not rearranging and effectively restarting by using parts. Note that substitutions could be used. $a=1\Rightarrow x\,\text{arcosh}(5x)-\int\frac{x}{\sqrt{25x^2-1}}dx\Rightarrow x\,\text{arcosh}(5x)-\frac{1}{25}\left(25x^2-1\right)^{\frac{1}{2}}\ (+c)$ $b=5\Rightarrow x\,\text{arcosh}(5x)-\int\frac{5x}{\sqrt{5x^2-1}}dx\Rightarrow x\,\text{arcosh}(5x)-\left(5x^2-1\right)^{\frac{1}{2}}\ (+c)$ $a=-5\Rightarrow x\,\text{arcosh}(5x)+\int\frac{5x}{\sqrt{25x^2-1}}dx\Rightarrow x\,\text{arcosh}(5x)+\frac{1}{5}\left(25x^2-1\right)^{\frac{1}{2}}\ (+c)$		
	$\int_{\frac{1}{4}}^{\frac{3}{5}}\text{arcosh}\,5x\,dx=\frac{3}{5}\text{arcosh}(3)-\frac{1}{5}\sqrt{25\times\frac{9}{25}-1}-\left(\frac{1}{4}\text{arcosh}\left(\frac{5}{4}\right)-\frac{1}{5}\sqrt{25\times\frac{1}{16}-1}\right)$ Applies appropriate limits (note substitutions above) with subtraction the right way round seen to obtain an expression of the form $x\,\text{arcosh}(5x)\pm f(x)$ **where f(x) has come from integration**		M1
	$=\frac{3}{5}\text{arcosh}(3)-\frac{2\sqrt{2}}{5}-\frac{1}{4}\text{arcosh}\left(\frac{5}{4}\right)+\frac{3}{20}$	Correct answer seen in any form. **Must not follow clearly incorrect work.**	A1
	$\text{arcosh}3=\ln\left(3+\sqrt{3^2-1^2}\right)$ or $\text{arcosh}\left(\frac{5}{4}\right)=\ln\left(\frac{5}{4}+\sqrt{\left(\frac{5}{4}\right)^2-1^2}\right)$ $\left\{\Rightarrow\frac{3}{5}\ln\left(3+\sqrt{8}\right)-\frac{2\sqrt{2}}{5}-\frac{1}{4}\ln 2+\frac{3}{20}\right\}$	Converts $\text{arcosh}(3)$ or $\text{arcosh}\left(\frac{5}{4}\right)$ to any correct log form. Independent mark but must have obtained $x\,\text{arcosh}(5x)\pm f(x)$ **where f(x) has come from integration**	M1
	$=\frac{3}{20}-\frac{2\sqrt{2}}{5}+\ln\left(3+2\sqrt{2}\right)^{\frac{3}{5}}-\frac{1}{4}\ln 2$ **Must not follow clearly incorrect work.**	Correct answer. Terms in any order but otherwise written as shown. Allow values for $p,\ q,\ r$ & k	A1
			(8)
			Total 10

	PAPER TOTAL: 75